from this cause particularly in the case of a potentially unstable device.

Conditions have also been obtained for the existence of points in the S_g plane for a given G_u corresponding to equality of gain figures for the "true" and "ideal" devices. The contour techniques and expressions given in this paper permit ready evaluation of the possible errors introduced by using some of the common "unilateral" design methods. With the assumption that $S_L = S_{22}^*$ it is possible to calculate such error bounds in terms of the transistor S-parameters and of the required gain. Use of these techniques thus provides an additional insight into the suitability of a transistor for a given application.

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Electrical Characteristics of Metal-Semiconductor Junctions

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Abstract—**The current–voltage characteristic of metal–semiconductor junctions is described by a simple equation which is the product of an exponential and a hyperbolic sine function if one includes the effect of tunneling. In the case of equal current flow via tunneling and via thermionic emission the conductance becomes a hyperbolic cosine function of the applied voltage. Devices displaying such characteristics appear attractive for use in harmonic frequency converters.**

I. Introduction

THE CARRIER transport mechanisms which determine the electrical properties of metal–semiconductor junctions have been investigated theoretically and experimentally since the early work of Braun [1]. Various types of junctions were named after the major investigators who made new contributions toward the understanding of the rectification process, typical examples being the Schottky diode [2] and the Mott diode [3]. A common characteristic of both these junctions is that the current is an exponential function of the applied voltage with the easy direction of electron flow being from the semiconductor to the metal.

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The purpose of this paper is to present equations for the current, the conductance, and the shot noise of metal–semiconductor junctions that encompass both a forward and a reversed direction of easy current flow. A reverse direction of rectification was already forseen by Wilson in 1932 [4] who postulated that the major mechanism of current transport was quantum-mechanical tunneling of electrons from the metal into the semiconductor through a thin interfacial barrier. It was not possible to fabricate such junctions for several decades. However, recent advances by Cho [5] in growing extremely thin MBE layers make it possible to achieve a controlled ratio of the current flow via tunneling and via thermionic emission. Junctions of this type appear useful for the development of hybrid integrated frequency converters because the required pump frequency can be a subharmonic of that which is used in conventional mixers and upconverters.

II. Current–Voltage Characteristic

The analysis of scattering and transport mechanisms of the carriers in a metal–semiconductor junction is rather complicated as shown in a recent paper by Salardi *et al.* [6]. There are, however, some simplifications possible

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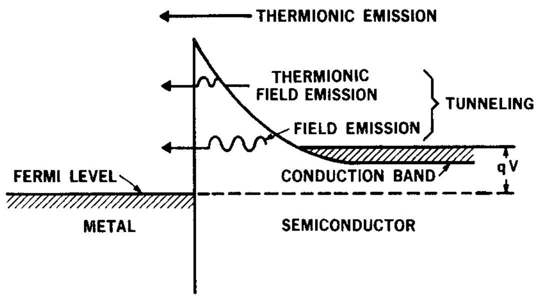

Fig. 1. Transport mechanisms of electrons across a metal–semiconductor barrier. The junction is at forward bias and the doping concentration of donors in the semiconductor is sufficiently high to cause degeneracy.

which describe the electron transport with sufficient accuracy for most practical applications such as switching and frequency conversion. A common feature of these simplified models is that the externally measured current is the sum of a thermionic emission current and a tunneling current, both currents being an exponential function of the externally applied voltage V. The various currents flowing across a forward biased barrier are shown in the schematic band diagram of Fig. 1. An equation for the current–voltage characteristic in terms of a sum of two exponentials was published in 1932 by Wilson [4]. The external current i was given by

$$i = \text{const}\left[\exp\left(\alpha\frac{qV}{kT}\right) - \exp\left\{(\alpha-1)\frac{qV}{kT}\right\}\right] \tag{1}$$

where q is the electron charge, k the Boltzmann constant, T the physical temperature of the junction, and α a parameter which is independent of voltage and lies between 1/2 and 1.

A similar equation based on a much more advanced understanding of current transport mechanisms was recently derived by Rideout and Crowell [7], [8]. Again the current is a sum of two exponential terms and given by

$$i = I_S\left[\exp\left(\frac{qV}{nkT}\right) - \exp\left\{\left(\frac{1}{n}-1\right)\frac{qV}{kT}\right\}\right] \tag{2}$$

where I_S is a saturation current and n an ideality factor which is equal or larger than 1. The major part of the first term in (2) is caused by thermionic emission, that means spilling of thermally excited electrons over the potential barrier. The second term, which is a reverse flux, is mainly due to tunneling which is composed of field emission and thermionic field emission. For the special case $n=1$, which represents a diode with negligible tunneling, one obtains the diode equation which is described in the well-known treatise by Sze [9]

$$i = I_S\left[\exp\left(\frac{qV}{kT}\right) - 1\right]. \tag{3}$$

The saturation current I_S is a function of the diode area S, the barrier height Φ_B, and the modified Richardson constant A^{**}. It is given by

$$I_S = A^{**}ST^2\exp\left\{\frac{q\Phi_B}{kT}\right\} \tag{4}$$

where $\Phi_B = kT/q + V_D + \Phi_F - \Delta\Phi$. V_D is the diffusion voltage, Φ_F the position of the Fermi level relative to the bottom of the conduction band, and $\Delta\Phi$ the image force lowering of the barrier.

Both (1) and (2) can be simplified if we introduce a new diode factor m defined by

$$m = \frac{2}{n} - 1 = 2\alpha - 1 \tag{5}$$

and a normalized voltage u

$$u = \frac{qV}{2kT}. \tag{6}$$

Equations (1) and (2) now become

$$i = I_S[\exp\{(m+1)u\} - \exp\{(m-1)u\}] \tag{7}$$

which can be written in the product form

$$i = 2I_S\exp(mu)\sinh u. \tag{8}$$

For the special case $m=0$ which represents a balance between tunneling and thermionic emission one obtains

$$i = 2I_S\sinh u. \tag{9}$$

A plot of the natural logarithm of the normalized current, $\ln(i/I_S)$, as a function of the normalized voltage is shown in Fig. 2. The same current–voltage characteristic is shown in Fig. 3 for positive and negative voltages. For $u \gg 1$ all traces are straight lines and for $m=0$ the current becomes an odd function of the applied voltage as stated in (9).

It is to be noted that (1) and (2) are not fulfilled for all metal–semiconductor junctions. For the general case the thermionic emission current i_{THERM} can be written in the form

$$i_{\text{THERM}} = A\{\exp(\alpha V) - 1\}. \tag{10}$$

The tunneling current is a reverse current if the semiconductor is highly doped or if the semiconductor consists of a highly doped substrate which is covered by a very thin epitaxial layer with a lower doping concentration. The reverse tunneling current is given by

$$i_{\text{TUNNEL}} = B\{1 - \exp(-\beta V)\}. \tag{11}$$

The total current i is given by

$$i = i_{\text{THERMAL}} + i_{\text{TUNNEL}}. \tag{12}$$

The saturation currents A and B depend on the height and the shape of the barrier. They can be made equal by making a linear voltage transformation $V' = V - V_0$, i.e., by applying a dc bias V_0 to the junction. The remaining coefficients α and β can be made equal by choosing an appropriate junction temperature or barrier profile. It is therefore possible to approach the current–voltage characteristic of (8) or (9) by proper choice of the materials, the doping profile of the semiconductor, and the junction temperature.

Junctions displaying the characteristics of (8) or (9) can also be fabricated using heterostructures such as a $Ga_{1-x}Al_xAs$ layer sandwiched between two GaAs layers.

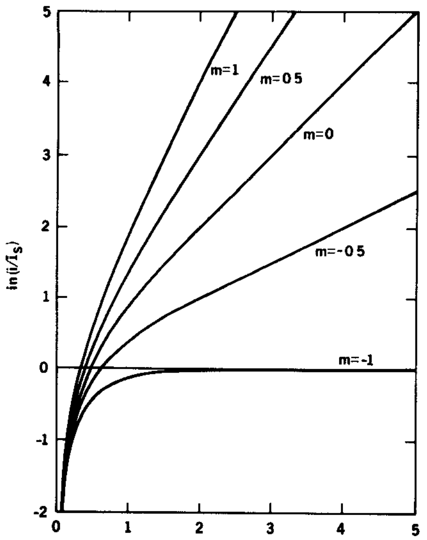

Fig. 2. Current–voltage characteristics at forward bias. The natural logarithm of the normalized current $i_n = i/I_S$ is plotted as a function of the normalized forward voltage $u = qV/(2kT)$.

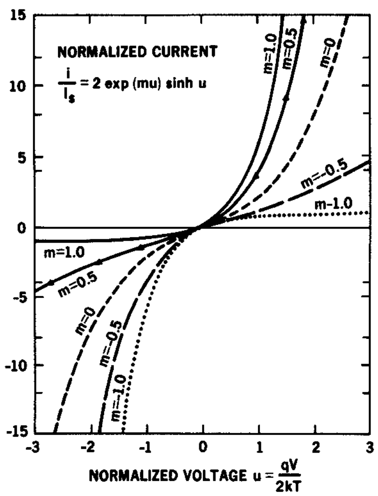

Fig. 3. Current–voltage characteristics at forward and at back bias. The curve labeled $m=1$ applies to pure thermionic emission and $m=-1$ to pure tunneling. A symmetrical hyperbolic sine function is obtained for a diode factor $m=0$.

Such a structure has been developed by Allyn, Gossard, and Wiegmann [10]. Because it is a special case of the heterjunction superlattice devised by Dingle [11], the device has been termed the "Single-Dingle Diode." The ratio of thermionic to tunneling current in this diode can be accurately controlled by a proper choice of the thickness of the $Ga_{1-x}Al_xAs$ layer and the profile of the index x, with $x=0$ at both boundaries of the layer.

III. Device Conductance

The conductance is an important electrical property which determines the suitability of a junction for electrical engineering applications. The conductance g is

$$g = \frac{\partial i}{\partial V} = \frac{q}{2kT}\frac{\partial i}{\partial u}. \tag{13}$$

From (7) we obtain

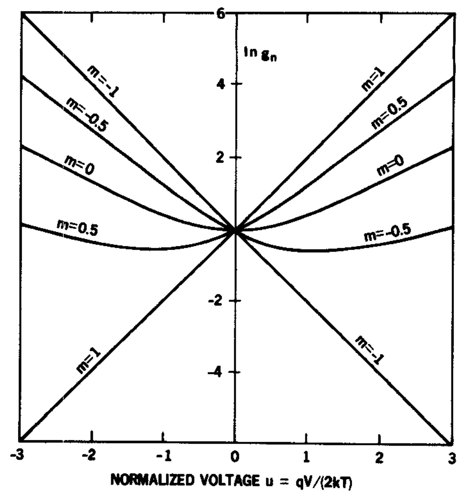

Fig. 4. Plot of the natural logarithm of the normalized device conductance, $g_n = kTg/(qI_S)$, as a function of the normalized voltage $u = qV/(2kT)$. Straight lines are obtained for limiting cases $m = \pm 1$.

TABLE I
Electrical Characteristics

Diode Factor m	Ideality Factor n	Normalized Current $i_n = i/(2I_S)$	Normalized Conductance $g_n = kTg/(gI_S)$
$+1$	1	$\exp u \sinh u$	$\exp(2u)$
0	2	$\sinh u$	$\cosh u$
-1	∞	$\exp(-u)\sinh u$	$\exp(-2u)$

$$g = \frac{qI_S}{kT}\exp(mu)\{m\sinh u + \cosh u\}. \tag{14}$$

For the special case $m=0$ this is reduced to

$$g = \frac{qI_S}{kT}\cosh u. \tag{15}$$

From (8) and (14) one obtains

$$g = \frac{q}{2kT}\left[(m+1)i + 2I_S\exp\{(m-1)u\}\right] \tag{16}$$

which for $i \gg I_S$ and $m=1$ gives the familiar result $g = qi/(kT)$.

Table I lists the ideality factor, the current, and the conductance for $m = \pm 1$ and $m=0$ which are the three cases of major interest. A normalized current, $i_n = i/2I_S$, and a normalized conductance

$$g_n = \frac{kT}{qI_S}g \tag{17}$$

are used to simplify the table. A plot of the natural logarithm $\ln g_n$, as a function of the normalized voltage is shown in Fig. 4. Straight lines with a slope of $\pm 45°$ are obtained for the limiting case $m = \pm 1$. The conductance for any intermediate case lies within the bounds of these straight lines.

IV. Shot Noise

The shot noise which is generated at the junction determines the minimum noise temperature which can be achieved in a frequency converter. A unified theory of high-frequency noise in Schottky type diodes has already been published by Viola and Mattauch [12]. We can extend this theory by writing (6) in the form

$$i=I_1-I_2=I_S\exp(mu)\{\exp(u)-\exp(-u)\} \quad (18)$$

where I_1 is mainly thermionic and I_2 mainly tunneling current flowing through the junction. In order to calculate the shot noise we have to add the currents I_1 and I_2 since they generate noise independently

$$\overline{\Delta i^2}=2q(I_1+I_2)\Delta f \quad (19)$$

where Δf is the bandwidth which is determined by the transmission properties of the external embedding network of the metal–semiconductor junction. The available shot noise power P_{SHOT} is

$$P_{\mathrm{SHOT}}=\frac{\overline{\Delta i^2}}{4g}=\frac{q}{2g}(I_1+I_2)\Delta f \quad (20)$$

and from (18)

$$P_{\mathrm{SHOT}}=\frac{qI_S}{g}\exp(mu)\cosh u\Delta f. \quad (21)$$

The final result using (14) is

$$P_{\mathrm{SHOT}}=\frac{kT}{1+m\tanh u}\Delta f. \quad (22)$$

The equivalent shot noise temperature of the junction is defined by

$$T_{\mathrm{eq}}=\frac{P_{\mathrm{SHOT}}}{k\Delta f}. \quad (23)$$

For $u\gg1$ and the special case $m=0$ we obtain from (22) and (23)

$$T_{\mathrm{eq}}=T. \quad (24)$$

For $u\gg1$ and pure thermionic emission with $m=1$ we obtain the well-known equation derived by Dragone [13] and Kerr [14]

$$T_{\mathrm{eq}}=\frac{T}{2}. \quad (25)$$

It is to be noted that there are additional noise mechanisms, such as intervalley scattering of the carriers, which increase the value of T_{eq} for both (24) and (25). The intervalley scattering contribution for $m=1$ will be larger compared to the case $m=0$ because electrons which tunnel through a barrier are not subject to this noise mechanism.

V. Frequency Conversion

The nonlinear characteristic of the junction can be used for frequency conversion by applying a pump frequency ω_p and a signal frequency ω_s to its terminals. Let us assume that the normalized pump voltage across the terminals of the junction is given by

$$u=U_P\cos\omega_P t \quad (26)$$

which means that the voltage is constrained to be sinusoidal, while the current i can have all the possible harmonics. The current and the conductance can be computed from the series expansions

$$\exp(z\cos\theta)=I_0(z)+2\sum_{k=1}^{\infty}I_k(z)\cos(k\theta) \quad (27)$$

$$\sinh(z\cos\theta)=2\sum_{k=1}^{\infty}I_{2k+1}(z)\cos\{(2k+1)\theta\} \quad (28)$$

$$\cosh(z\cos\theta)=I_0(z)+2\sum_{k=1}^{\infty}I_{2k}(z)\cos(2k\theta) \quad (29)$$

where I_k is the modified Bessel function of the first kind of order k. From (7), (26), and (27) one obtains

$$i=I_S\left[A_0+2\sum_{k=1}^{\infty}A_k\cos(k\omega_P t)\right] \quad (30)$$

$$A_k=I_k\{(m+1)U_P\}-I_k\{(m-1)U_P\} \quad (31)$$

and for the conductance g

$$g=\frac{qI_S}{2k}\left[B_0+2\sum_{k=1}^{\infty}B_k\cos(k\omega_P t)\right] \quad (32)$$

$$B_k=(m+1)I_k\{(m+1)U_P\}-(m-1)I_k\{(m-1)U_P\}. \quad (33)$$

The special case $m=0$ is of interest because $A_k=0$ for $k=0,2,4,\cdots$ and $B_k=0$ for $k=1,3,5,\cdots$. The resulting current and the conductance for $m=0$ are

$$i=4I_S\sum_{k=0}^{\infty}I_{2k+1}(U_P)\cos\{(2k+1)\omega_P t\} \quad (34)$$

$$g=\frac{qI_S}{kT}\left[I_0(U_P)+2\sum_{k=1}^{\infty}I_{2k}(U_P)\cos(2k\omega_P t)\right]. \quad (35)$$

From (34) and (35) one concludes 1) there is no dc current flowing through the junction, 2) the device current contains only odd order harmonics of the pump frequency, and 3) the conductance contains only even order harmonics.

These electrical properties of the pumped junction are thus identical to those of the two-diode frequency converter described by Carlson *et al.* [15]. As expected the device is therefore suitable for use in subharmonic mixers and subharmonic upconverters. The conversion characteristics can be obtained using the harmonic expansion from g of (35) and calculating the conversion loss as described extensively in the standard work by Saleh [16].

VI. Conclusions

The current, conductance, and shot noise of metal–semiconductor junctions with electron transport via tunneling are described by simple equations. An important parameter in these equations is the diode factor m which determines the ratio of tunneling to thermionic emission. For $m=0$ the conductance becomes a hyperbolic cosine function and the properties of the device are comparable to the two-diode structure used in modern frequency converters.

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